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State Discontinuity Analysis of Linear Switched Systems via Energy Function Optimization

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***Abstract*—In this paper, the objects of study are electrical networks consisting of passive elements, independent sources, and ideal switches. We study the discontinuities in state variable that are caused by the switching behavior. The main contribution of the paper is to present an energy-based state jump rule that is equivalent to those based on charge/flux conservation principle and Laplace transform. The advantage of the proposed rule lies in the fact that one can explicitly compute the state jump.**

I. Introduction

The analysis of switching circuits is commonly carried out by approximating the ON state of the switch with a shortcut and the OFF state with an open circuit. Because of such approximations, the (instantaneous) commutations can lead to inconsistent initial conditions and hence, may cause discontinuity (*jump*) in the state variables. Finding the discontinuity due to a change of switch configuration (topology) raises as a natural issue. The main goal of the current paper is to introduce a state jump rule that yields the state at t_0^+ (t_0 being the switching instant) in an explicit manner depending on the given state at t_0^- and the new switch configuration. The early work on this topic was presented in [1] where the author introduced the Dirac impulse (and its derivatives) in the analysis of linear active RLC networks and gave an iterative method to obtain a set of algebraic equations due to capacitor loops, inductor cut-sets and/or to degeneracies caused by active elements. In case of passivity, the proposed method is analogous to the charge/flux conservation approach [2], [3]. In [4] the principle of charge/flux conservation was used to obtain consistent initial conditions at t_0^+. The method is applied to periodically operated switched networks, but the author did not present a systematic approach to compute the consistent states. Later, in [5] an iterative method was presented based on the so-called switching transformation matrix which expresses discontinuities of the state variables at the switching instances. A distributional framework was used in [6], whose framework does not include current sources. The papers [7], [8] investigated and gave an interpretation of the possible energy loss at the switching. All above mentioned studies work within the state-space framework. Another line of research was developed by employing Laplace transformation techniques. For instance, numerical inversion of Laplace transformation was used in [9], [10]. The same authors proposed a method based on graphs [11] and the applications of the charge/flux principles allowed to obtain consistent initial conditions t_0^+. The case of periodically switched nonlinear circuits was considered in [12]. In all these works the connection between the Laplace transform and consistent initial conditions is not well-established. Numerical computation of the state jump was also the subject of [13], [14], [15] and [16]. Nonlinear switching circuits were considered in [17] and [18] where an a-priori knowledge of the circuit behaviour across the switching instant is needed.

In spite of the wide literature on the topic, none of the proposed techniques yields an explicit expression for the state jump. The main contribution of the paper is the derivation of an explicit expression for the state discontinuity in linear passive switched systems. Our approach is inspired by the work presented in [19]. Our treatment does not require fixing the switch configuration. The discontinuity (jump) in the state variable produced by the instantaneous change in the network topology is obtained as the solution of a constrained energy based minimization problem. We complete the characterization by showing that this jump rule is equivalent to the application of charge/flux conservation principle and to results obtained by Laplace transform techniques. Thus, the connection between the Laplace transform techniques and consistent initial conditions is fully exhibited.

II. Linear Switched Systems (LSS)

Consider an electrical network containing linear passive elements (such as resistors, capacitors, inductors, transformers, and gyrators), sources, and ideal switches. Under mild conditions (see [20]), its dynamics can be given as

$$\frac{d}{dt}x(t) = Ax(t) + Bz(t) + Eu(t), \quad x(0) = x_0 \tag{1a}$$

$$w(t) = Cx(t) + Dz(t) + Fu(t) \tag{1b}$$

where x is the state, x_0 the initial state, u the vector of sources, and (z, w) the *switch variables*, i.e.

$$\text{either } z_i(t) = 0 \text{ or } w_i(t) = 0 \tag{1c}$$

for each time instant $t \geqslant 0$. We call such systems *linear switched systems* (LSS).

We say that LSS (1) is in the *switch configuration* $\pi \subseteq \{1, 2, \ldots, m\}$ on some time interval if

$$w_i(t) = 0 \qquad \text{if } i \in \pi \tag{2a}$$
$$z_i(t) = 0 \qquad \text{if } i \notin \pi \tag{2b}$$

for all time instants t in the same interval.

As an example, consider the circuit depicted in Figure 1. Its dynamics can be described by the LSS

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} i_{L_1} \\ i_{L_2} \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ 0 & -\frac{R}{L_2} \end{bmatrix}\begin{bmatrix} i_{L_1} \\ i_{L2} \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{L_1} \\ 0 & \frac{1}{L_2} \end{bmatrix}\begin{bmatrix} v_{S_1} \\ v_{S_2} \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & 0 \\ \frac{1}{L_2} & 0 \end{bmatrix}\begin{bmatrix} u_E \\ u_J \end{bmatrix} \end{aligned} \tag{3a}$$

$$\begin{bmatrix} i_{S_1} \\ i_{S_2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} i_{L_1} \\ i_{L_2} \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} u_E \\ u_J \end{bmatrix} \tag{3b}$$

$$\text{either } i_{S_1} = 0 \text{ or } v_{S_1} = 0 \tag{3c}$$
$$\text{either } i_{S_2} = 0 \text{ or } v_{S_2} = 0. \tag{3d}$$

As the circuit contains two switches, there are four possible switch configurations:

1) S_1 is OFF and S_2 is OFF. The switch configuration is $\pi = \{1, 2\}$, i.e. $w_1 = i_{S_1} = 0$ and $w_2 = i_{S_2} = 0$.
2) S_1 is OFF and S_2 ON. The switch configuration is $\pi = \{1\}$, i.e. $w_1 = i_{S_1} = 0$ and $z_2 = v_{S_2} = 0$.
3) S_1 is ON and S_2 is OFF. The switch configuration is $\pi = \{2\}$, i.e. $z_1 = v_{S_1} = 0$ and $w_2 = i_{S_2} = 0$.
4) S_1 is ON and S_2 is ON. The switch configuration is $\pi = \varnothing$, i.e. $z_1 = v_{S_1} = 0$ and $z_2 = v_{S_2} = 0$.

Given a switch configuration $\pi \subseteq \{1, 2, \ldots, m\}$, the dynamics of the LSS (1) are described by the differential-algebraic equations (DAEs)

$$\frac{d}{dt}x(t) = Ax(t) + B_{\bullet\pi}z_\pi(t) + Eu(t), \quad x(0) = x_0 \tag{4a}$$
$$0 = C_{\pi\bullet}x(t) + D_{\pi\pi}z_\pi(t) + F_{\pi\bullet}u(t) \tag{4b}$$
$$w_{\pi^c}(t) = C_{\pi^c\bullet}x(t) + F_{\pi^c\bullet}u(t). \tag{4c}$$

where π^c denotes the set complement of π and for instance $C_{\pi\bullet}$ denotes the submatrix of C consisting only the rows indexed by π. Obviously, the equation (4b) may impose constraints on the external inputs u and/or initial states x_0. This observation leads to the questions:

1) For which inputs do these DAEs admit a solution?
2) For which initial states do these DAEs admit a solution?

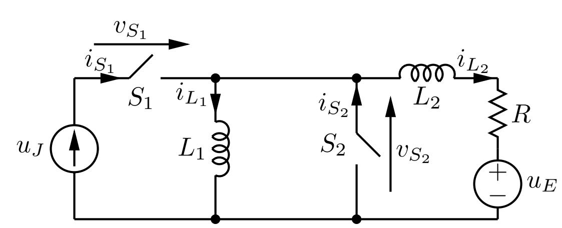

Figure 1. A switched network with possible inconsistent initial condition, e.g. $i_{L_1} = 0$ and $i_{L_2} = 0$ when $\pi = \{2\}$.

In the case these DAEs do not admit a solution for an input and/or initial state, it is natural to ask:

3) What could be a physical interpretation of non-existence of solutions?

The last question differs from the previous two in nature. Indeed, determining 'admissible' inputs and 'consistent' initial states become a mathematical problem once the 'solution' is precisely defined. However, the last question involves the physics of the system at hand.

Definition II.1 We say that a triple (z, x, w), where x is absolutely continuous and (z, w) are locally square integrable, is a solution of (1) for the initial state x_0 and the Bohl-type (see [19]) input u if $x(0) = x_0$ and (1) is satisfied almost everywhere.

Note that the above definition of solution does not allow state jumps.

Definition II.2 We say that

- an input u is *admissible* with respect to (w.r.t.) the switch configuration π if it is Bohl type and the DAEs (4) admit a solution at least for an initial state x_0.
- an initial state x_0 is *consistent* w.r.t. the switch configuration π and the admissible input u if the DAEs (4) admit a solution for the initial state x_0 and the input u.

Consider the circuit in Figure 1 with $\pi = \{2\}$. Equation (4b) becomes $0 = i_{L_1}(t) + i_{L_2}(t) - u_J(t)$. Thus given the input u_J, the consistent initial conditions must satisfy $i_{L_1}(0) + i_{L_2}(0) = u_J(0)$. The following theorem characterizes admissible inputs and consistent states.

Theorem II.3 *Consider the LSS* (1). *Suppose that the system* $\Sigma(A, B, C, D)$ *is passive. Then the following statements hold.*

1) *An input u is admissible w.r.t. the switch configuration π if, and only if,*
$$F_{\pi\bullet}u(t) \in \operatorname{im}\begin{bmatrix} C_{\pi\bullet} & D_{\pi\pi} \end{bmatrix} \textit{ for all } t \geqslant 0. \tag{5}$$
2) *An initial state x_0 is consistent w.r.t. the switch configuration π and the admissible input u if, and only if,*
$$C_{\pi\bullet}x_0 + F_{\pi\bullet}u(0) \in \operatorname{im} D_{\pi\pi}. \tag{6}$$

Proof. The proof readily follows by applying Theorem A.6 item 2 in [21]. ■

III. Energy Based Jump Rule

When the initial state is not consistent with respect to a given switch configuration and input, it is natural to consider a jump (discontinuity) in the state variable such that the re-initialized state is consistent. Computation of the re-initialized state has been extensively studied in the literature. Roughly speaking, there are two main approaches that are based on charge/flux conservation principle and Laplace transform techniques. In what follows, we propose an alternative method that is based on energy minimization principle. Later, we will show that this jump rule is equivalent to charge/flux conservation rule, as well as the Laplace transform method, for linear passive electrical networks. The advantage of our method is, however, the fact that an explicit expression of the state jump can be obtained.

Consider the LSS (1) where $\Sigma(A,B,C,D)$ is passive. It can be shown [19, Prop. 2.4] that this is equivalent to the existence of a solution K of the linear matrix inequalities (LMIs)

$$K = K^T \geqslant 0 \tag{7a}$$

$$\begin{bmatrix} A^TK + KA & KB - C^T \\ B^TK - C & -(D+D^T) \end{bmatrix} \leqslant 0. \tag{7b}$$

Suppose that the LMIs (7) admit a positive definite solution K. Let $x(0^-)$ be the initial state, π be a switch configuration at 0^+ and u be an admissible input. Consider the minimization problem (with respect to $x(0^+)$)

$$\text{minimize} \frac{1}{2}(x(0^+) - x(0^-))^T K (x(0^+) - x(0^-)) \tag{8a}$$

$$\text{subject to } C_{\pi\bullet}x(0^+) + F_{\pi\bullet}u(0) \in \operatorname{im} D_{\pi\pi}. \tag{8b}$$

For a given positive definite K, as the constraints are linear, this problem admits a unique solution [22]. Although (7) by itself might have non-unique solutions, it can be shown that the solution of (8) is independent of the choice of K (see [21]), provided K satisfies LMIs (7). We take the unique solution $x(0^+)$ of (8) as the re-initialized state.

Consider the minimization problem (8). For notational simplicity, let $\bar{C} = C_{\pi\bullet}$, $\bar{D} = D_{\pi\pi}$ and $\bar{F} = F_{\pi\bullet}$. Let P be a matrix with full row rank such that $\ker P = \operatorname{im} \bar{D}$ then, it holds that

$$\begin{aligned} \bar{C}x(0^+) + \bar{F}u(0) \in \operatorname{im} \bar{D} &\Leftrightarrow P(\bar{C}x(0^+) + \bar{F}u(0)) = 0 \\ &\Leftrightarrow P\bar{C}(x(0^+) - x(0^-)) = -P\bar{F}u(0) - P\bar{C}x(0^-) \end{aligned} \tag{9}$$

Now let $\Phi = P\bar{C}$, $\beta = -P\bar{F}u(0) - P\bar{C}x(0^-)$ and $\xi = x(0^+) - x(0^-)$, the problem (8) can be rewritten as

$$\text{minimize } \frac{1}{2}\xi^T K \xi \tag{10a}$$

$$\text{subject to } \Phi\xi = \beta. \tag{10b}$$

This problem can be solved by forming the Lagrangian

$$L = \frac{1}{2}\xi^T K \xi + \lambda^T(\Phi\xi - \beta)$$

then by setting its derivatives with respect to ξ and λ to zero one can obtain the following set of linear equations

$$0 = \frac{\partial L}{\partial \xi} = K\xi + \Phi^T\lambda \tag{11a}$$

$$0 = \frac{\partial L}{\partial \lambda} = \Phi\xi - \beta. \tag{11b}$$

From (11a), we get

$$\xi = -K^{-1}\Phi^T\lambda. \tag{12}$$

By substituting ξ into (11b), we get

$$\lambda = -(\Phi K^{-1}\Phi^T)^\dagger\beta \tag{13}$$

where $(\Phi K^{-1}\Phi^T)^\dagger$ is the Moore-Penrose generalized inverse of $\Phi K^{-1}\Phi^T$ (see e.g. [23]). Finally, the solution of the problem (10) is

$$\xi = K^{-1}\Phi^T(\Phi K^{-1}\Phi^T)^\dagger\beta. \tag{14}$$

Thus the solution of the problem (8) is

$$\begin{aligned} x(0^+) &= x(0^-) + K^{-1}\Phi^T(\Phi K^{-1}\Phi^T)^\dagger\beta \\ &= (I - K^{-1}\bar{C}^TP^TQ^\dagger P\bar{C})x(0^-) \\ &\quad -K^{-1}\bar{C}^TP^TQ^\dagger P\bar{F}u(0) \end{aligned} \tag{15}$$

where $Q = \Phi K^{-1}\Phi^T = P\bar{C}K^{-1}\bar{C}^TP^T$.

Note that in (8) the matrix K depends only on the system matrices (A,B,C,D), while other matrices depend on the actual switch configuration. If the matrix $\bar{D}$ is invertible then the constraint (8b) is satisfied by $x(0^+) = x(0^-)$ for any input u. Hence, the unique solution of (8) is given by $x(0^+) = x(0^-)$ for any input u. In other words, all inputs are admissible for the corresponding switch configuration and all initial states are consistent.

Another common method to solve the state jump problem is to use Laplace transforms. For a given switch configuration π, one can take the Laplace transform of (4). This yields

$$\begin{aligned} x(s) &= (sI - A)^{-1}x_0 + (sI - A)^{-1}B_{\bullet\pi}z_\pi(s) \\ &\quad +(sI - A)^{-1}Eu(s) \end{aligned} \tag{16a}$$

$$\begin{aligned} 0 &= C_{\pi\bullet}(sI - A)^{-1}x_0 \\ &\quad +[D_{\pi\pi} + C_{\pi\bullet}(sI - A)^{-1}B_{\bullet\pi}]z_\pi(s) \\ &\quad +[F_{\pi\bullet} + (sI - A)^{-1}E]u(s) \end{aligned} \tag{16b}$$

$$\begin{aligned} w_{\pi^c} &= C_{\pi^c\bullet}(sI - A)^{-1}x_0 + C_{\pi^c\bullet}(sI - A)^{-1}B_{\bullet\pi}z_\pi(s) \\ &\quad +[F_{\pi^c\bullet} + C_{\pi^c\bullet}(sI - A)^{-1}E]u(s). \end{aligned} \tag{16c}$$

For an initial state x_0 and input u, one looks for the solution of (16). According to Theorem A.6 item 1 in [21], there always exist solutions to (16) if the underlying system $\Sigma(A,B,C,D)$ is passive and u is admissible. The following theorem links the solution of (16) to our energy based jump rule.

Theorem III.1 *Suppose that $\Sigma(A,B,C,D)$ is passive, the LMIs (7) admit a positive definite solution and*

$$F_{\pi\bullet}u(t) \in \operatorname{im} \begin{bmatrix} C_{\pi\bullet} & D_{\pi\pi} \end{bmatrix}$$

for all $t \geqslant 0$. Let $z_\pi(s)$ be a proper solution of (16) with $x_0 = x(0^-)$ and $x(0^+)$ be the solution of the minimization problem (8). Then,

$$x(0^+) = x(0^-) + Bz_0$$

where z_0 is such that $Bz_0 = \lim_{s\to\infty} Bz_\pi(s)$.

Proof. See [21] ■

We now show the equivalence between the energy based rule and the charge/flux conservation rule on an example. Consider the circuit depicted in Figure 1. This circuit is the same as the one given in [2, Section 4.3], if the switch configuration $\pi = \{2\}$ is considered. Note that $D = 0$ hence it can be chosen $P = 1$ (or any nonzero real number). By considering at 0^+ $\pi = \{2\}$, we get

$$C_{2\bullet} = \begin{bmatrix} 1 & 1 \end{bmatrix}, F_{2\bullet} = \begin{bmatrix} 0 & -1 \end{bmatrix}, \tag{17a}$$

$$\Phi = C_{2\bullet},\, K = \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix}, \tag{17b}$$

where K is the matrix used in the storage function. The re-initialized state can be easily obtained from equation (15)

$$i_{L_1}(0^+) = \frac{L_1 i_{L_1}(0^-) - L_2 i_{L_2}(0^-) + L_2 u_J(0)}{L_1 + L_2} \tag{18a}$$

$$i_{L_2}(0^+) = \frac{-L_1 i_{L_1}(0^-) + L_2 i_{L_2}(0^-) + L_1 u_J(0)}{L_1 + L_2}. \tag{18b}$$

The results are the same as that obtained in [2], where the solution is obtained by using the principle of charge/flux conservation. Consider the circuit given in Figure 1 with the following parameters: $L_1 = L_2 = 1$mH, $R = 1\Omega$, $u_J(0) = 5$A, $i_{L_1}(0^-) = 1$A, $i_{L_2}(0^-) = 2$A. By using (18), it is simple to obtain $i_{L_1}(0^+) = 2$A and $i_{L_2}(0^+) = 3$A.

In order to get the numerical solution corresponding to the energy based jump rule the LMI (7) can be solved by using the LMIs MATLAB commands:

```
>>setlmis([]);
>>lmiterm([-2 1 1 K],1,1);     %(7a): K
>>K=lmivar(1,[length(A),1]);
>>lmiterm([1 1 1 K],A',1,'s');%(7b):A'K+KA
>>lmiterm([1 2 1 K],B',1);     %(7b):B'K
>>lmiterm([1 2 1 0],-C);       %(7b):-C
>>lmiterm([1 2 2 0],-D-D');    %(7b):-D-D'
>>passiveLMI=getlmis;
>>[tmin,Ksol]=feasp(passiveLMI);
>>K=dec2mat(passiveLMI,Ksol,1);
```

The solution provided by the previous commands is equal to the matrix K in (17b). The matrix P such that $\ker P = \operatorname{im} \bar{D}$ can be obtained by using the MATLAB command

```
>>P=null(Dbar')';
```

By using the pinv MATLAB command and using (15), the expected solution $i_{L_1}(0^+) = 2$ and $i_{L_2}(0^+) = 3$ is obtained with an accuracy of 10^{-10}.

IV. Conclusion and Future Work

A novel approach for the evaluation of the state jump for linear passive networks with ideal switches and sources has been proposed. The proposed procedure is based on the minimization of the energy associated with the state discontinuity at the switching instant and allows an explicit computation of the state jump. We have proved that the proposed energy-based jump rule is related to the Laplace transform. Then, we have illustrated by means of an example that the energy-based jump rule is equivalent to the application of charge/flux conservation principle. Hence, the relation between this principle and the Laplace transform is completely characterized.

Future work will deal with the application of the proposed technique to the case of Modified Nodal Analysis representation and with the numerical computation of state discontinuities.

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